On the energetics of the dyadotorus

Christian Cherubini*°, Andrea Geralico°, Jorge Rueda°, Remo Ruffini°.

*Universiy Campus Biomedico, Rome.
°International Centre for Relativistic Astrophysics, Rome

3rd Stueckelberg Workshop

Pescara, 2008

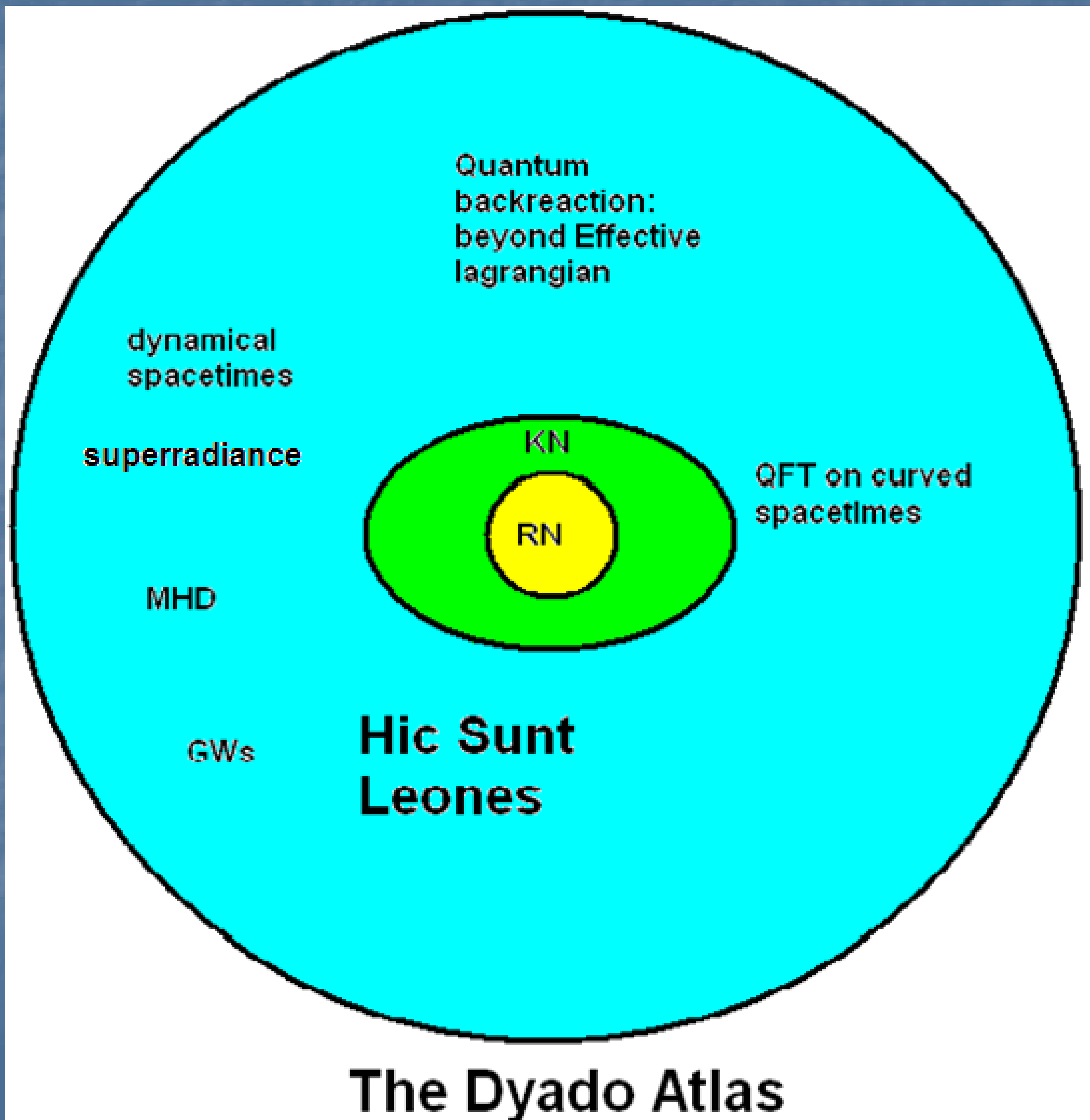
Flashback: ICRA Pescara, July, 2007.

"…There is a great deal of activity in the field these days, but this activity is mainly in showing that the previous activity of somebody else resulted in an error or in nothing useful or in something promising. It is like a lot of worms trying to get out of a bottle by crawling all over each other. It is not that the subjectis hard; it is that the good men are occupied elsewhere. Remind me not to come to any more gravity conferences…"

(R.P. Feynman, Warsaw Conference 1962)

We are going to talk about Energy in General Relativity: stars are in the sky, GR works fine, so we need a pragmatic approach to the problem.

Dyadoregion



$$|\mathbf{E}| = E_c = \frac{m_e^2 c^3}{\hbar e} \,,$$

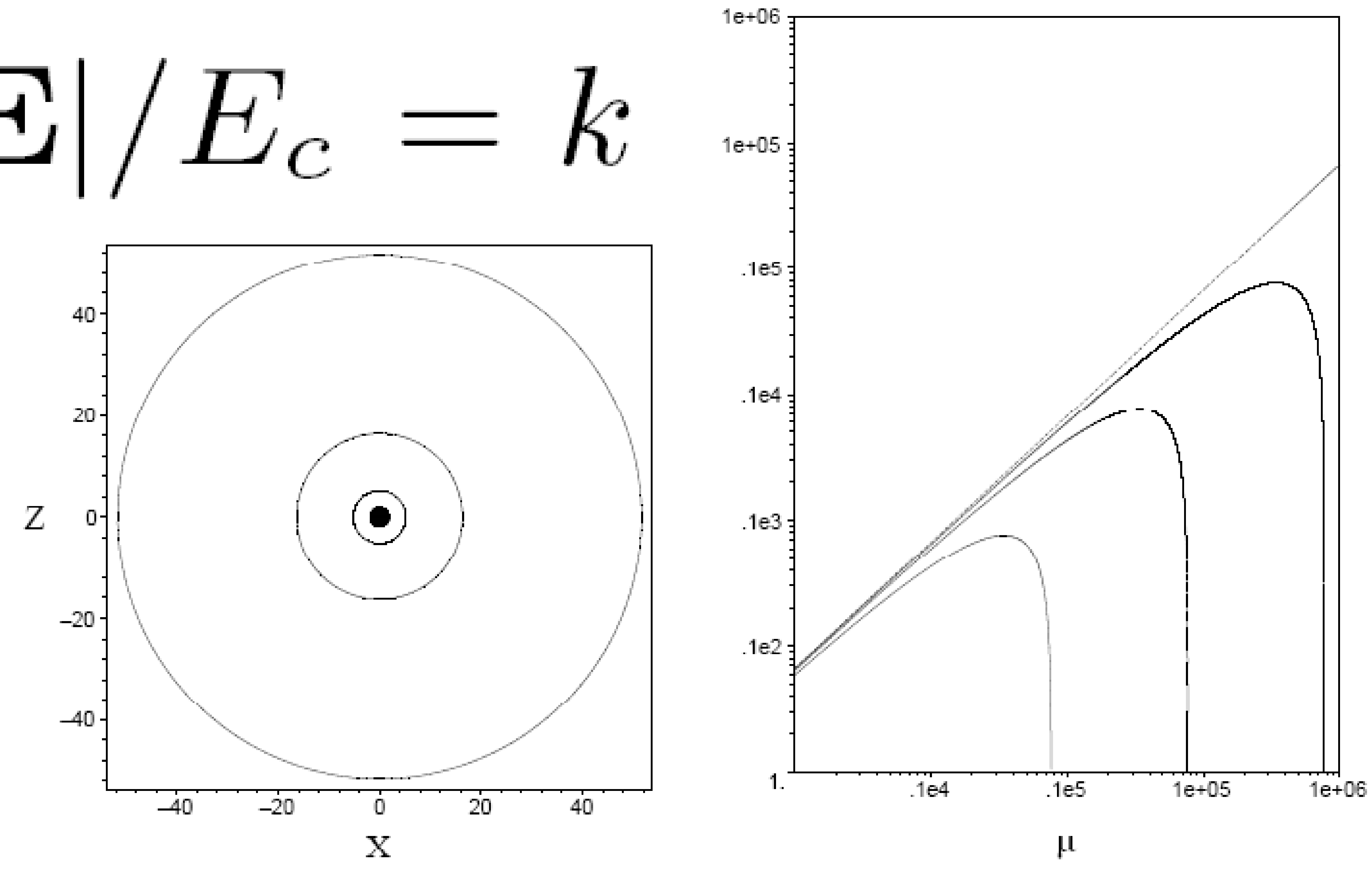
Reissner-Norstrom (Ruffini et al.)

$$r_{\rm ds} = 1.12 \times 10^8 \sqrt{\xi\mu}\ {\rm cm}$$

$$|\mathbf{E}|/E_c = k$$

$$\mu = M/M_\odot$$

$$\xi = Q/M$$



(a) (b)

FIG. 3: The projection of the dyadosphere on the $X - Z$ plane ($X = r\sin\theta$, $Z = r\cos\theta$ are Cartesian-like coordinates built up simply using the Boyer-Lindquist radial and angular coordinates) is shown in Fig. (a) for a non-extreme Reissner-Norström black hole with the choice of parameter $\mu = 10^3$ and $\xi = 0.5$ and different values of the ratio $|\mathbf{E}|/E_c = k = [0.1, 1, 10]$. The exterior curve corresponds to $k = 0.1$, the dyadosphere shrinking for increasing values of k, i.e. as the electric field strength becomes larger and larger than the critical field. The inner black disk represents the black hole horizon. Fig. (b) shows instead the behaviour of the electromagnetic energy (7) as a function of μ as shown in Fig. 1 for $\xi = 0.5$ and different values of $k = [0.1, 1, 10]$, moving from the right to the left.

Total energy of pairs

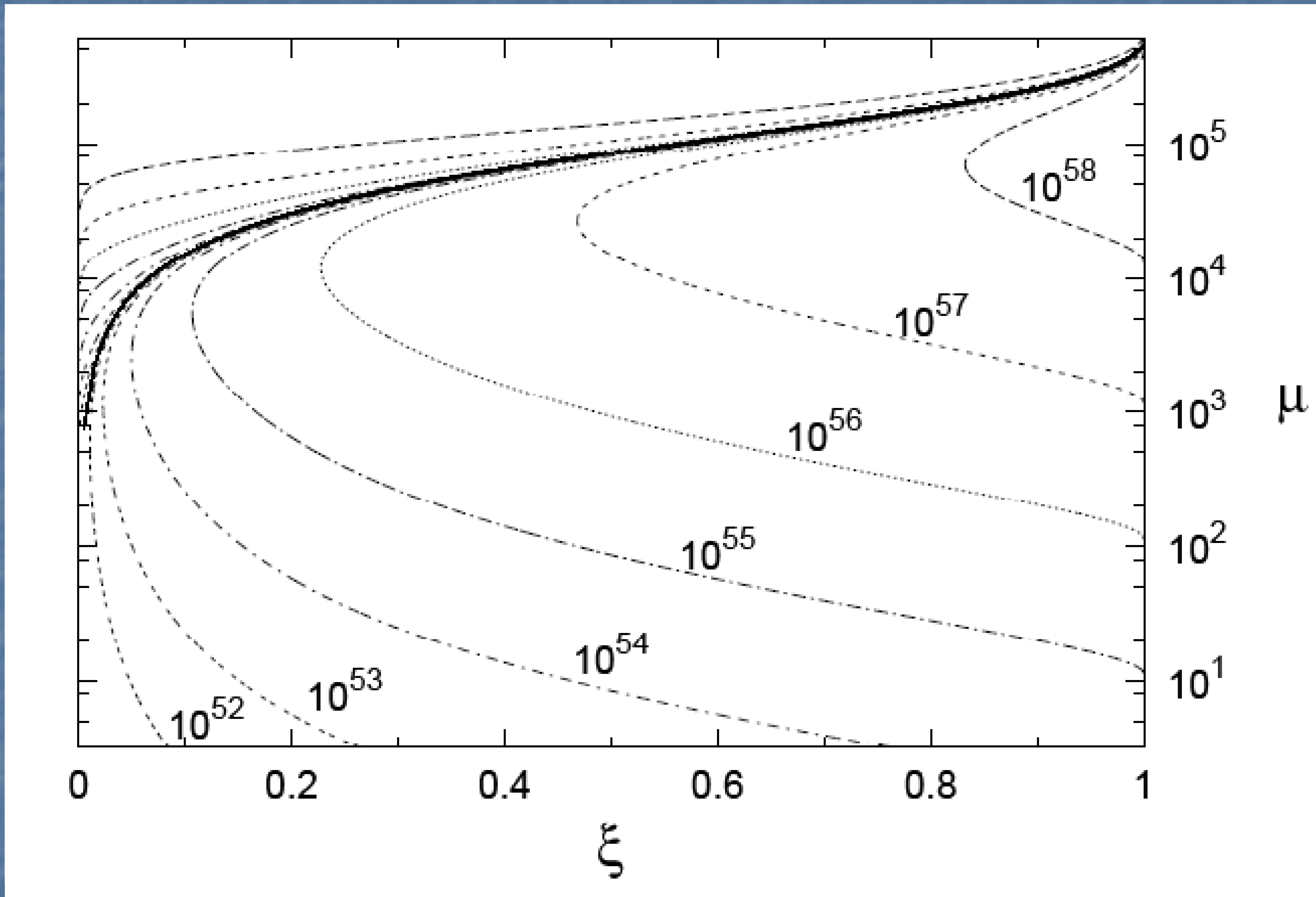


FIG. 2: Selected lines corresponding to fixed values of the the total energy of pairs (9) are given as a function of the two parameters μ and ξ. Only the solutions below the continuous heavy line are physically relevant. The configurations above the continuous heavy lines correspond to unphysical solutions with $r_{\rm ds} < r_+$.

Kerr-Newman: regular charged and rotating stationary and axisymmetric black hole.

Boyer-Lindquist coordinates, signature (+,-,-,-)

$$ds^2 = \left(1 - \frac{2Mr - Q^2}{\Sigma}\right) dt^2 + \frac{2a\sin^2\theta}{\Sigma}(2Mr - Q^2)\, \underline{dt\,d\phi} - \frac{\Sigma}{\Delta} dr^2$$
$$-\Sigma d\theta^2 - \left[r^2 + a^2 + \frac{a^2\sin^2\theta}{\Sigma}(2Mr - Q^2)\right]\sin^2\theta d\phi^2 \;,$$

$$F = \frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta) dr \wedge [dt - a\sin^2\theta d\phi]$$
$$+2\frac{Q}{\Sigma^2} ar\sin\theta\cos\theta d\theta \wedge [(r^2 + a^2)d\phi - a d\phi] \;.$$

$$\Sigma = r^2 + a^2\cos^2\theta \;\; \text{and} \;\; \Delta = r^2 - 2Mr + a^2 + Q^2.$$

Effective Lagrangian of QED

Remo Ruffini and She-Sheng Xue

arXiv:hep-th/0609081v1

Rapid Review of QFT side

Starting point: QED in external electromagnetic fields (i.e. EM field not quantized)

If a constant Electric field only is present

$$\frac{\Gamma}{V} = \frac{\alpha E^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{n\pi E_c}{E}\right),$$

decay rate of the vacuum per unit volume

If both constant Electric and Magnetic fields are present (Schwinger)

$$\frac{\Gamma}{V} = \frac{\alpha\varepsilon^2}{\pi^2}\sum_{n=1}\frac{1}{n^2}\frac{n\pi\beta/\varepsilon}{\tanh n\pi\beta/\varepsilon}\exp\left(-\frac{n\pi E_c}{\varepsilon}\right)$$

$$\varepsilon^2-\beta^2 \equiv \mathbf{E}^2-\mathbf{B}^2 \equiv 2S, \quad \varepsilon\beta \equiv \mathbf{E}\cdot\mathbf{B} \equiv P,$$

It can be convenient to go to the CF, but the *final result is expressed in terms of Maxwell invariants.*

$$\left\{\begin{array}{c}\varepsilon\\ \beta\end{array}\right\} \equiv \frac{1}{\sqrt{2}}\sqrt{\sqrt{(\mathbf{E}^2-\mathbf{B}^2)^2+4(\mathbf{E}\cdot\mathbf{B})^2}\pm(\mathbf{E}^2-\mathbf{B}^2)}$$
$$= \sqrt{(S^2+P^2)^{1/2}\pm S}. \quad (33)$$

Then exists a special Lorentz frame to be called the *center-of-fields frame*, where the fields $\mathbf{B}_{\rm CF}$ and $\mathbf{E}_{\rm CF}$ are parallel. In this frame, $\beta = |\mathbf{B}_{\rm CF}|$ and $\varepsilon = |\mathbf{E}_{\rm CF}|$.

Let's compute DyadoRegion fast (Newman-Penrose).

$$l^\mu = \frac{1}{\Delta}[r^2+a^2, \Delta, 0, a] \ ,$$
$$n^\mu = \frac{1}{2\Sigma}[r^2+a^2, -\Delta, 0, a] \ ,$$
$$m^\mu = \frac{1}{\sqrt{2}(r+ia\cos\theta)}\left[ia\sin\theta, 0, 1, \frac{i}{\sin\theta}\right]$$

$$\phi_0 = \phi_2 = 0 \ , \qquad \phi_1 = \frac{Q}{2}\rho^2 \ .$$
$$\psi_2 = M\rho^3 + Q^2\rho^*\rho^3 \ ,$$
$$\rho = -\frac{1}{r - ia\cos\theta}$$

Electrovac Petrov type D

(I) Maxwell Invariants in NP form

$$\mathcal{F}_1 \equiv \frac{1}{2}F_{\mu\nu}F^{\mu\nu} = \mathbf{B}^2 - \mathbf{E}^2 = 4\mathrm{Re}(\phi_0\phi_2 - \phi_1^2) \ ,$$
$$\mathcal{F}_2 \equiv \frac{1}{2}F_{\mu\nu}{}^*F^{\mu\nu} = 2\,\mathbf{E}\cdot\mathbf{B} = -4\mathrm{Im}(\phi_0\phi_2 - \phi_1^2) \ ,$$

(II) Parallel fields frame (Carter Frame)

$$|\mathbf{B}|^2 - |\mathbf{E}|^2 = -4\mathrm{Re}(\phi_1^2) \ , \qquad 2\,|\mathbf{E}|\,|\mathbf{B}| = 4\mathrm{Im}(\phi_1^2) \ ,$$

(III) Dyado-region KN

$$|\mathbf{E}| = \left|\frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta)\right| \ , \quad |\mathbf{B}| = \left|2\frac{Q}{\Sigma^2}ar\cos\theta\right| \ .$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

$$Q\,\frac{(r^2 - a^2\cos^2\theta)}{(r^2 + a^2\cos^2\theta)^2} = E_c$$

(Damour Ruffini '70s)

Outside the outer horizon for positive

Solving for r and introducing the dimensionless quantities $\xi = Q/M$, $\alpha = a/M$, $\mu = M/M_{\odot}$ and $\epsilon = E_c M_{\odot} \approx 1.873\times 10^{-6}$ (with $M_{\odot} \approx 1.477\times 10^{5}$ cm) we get

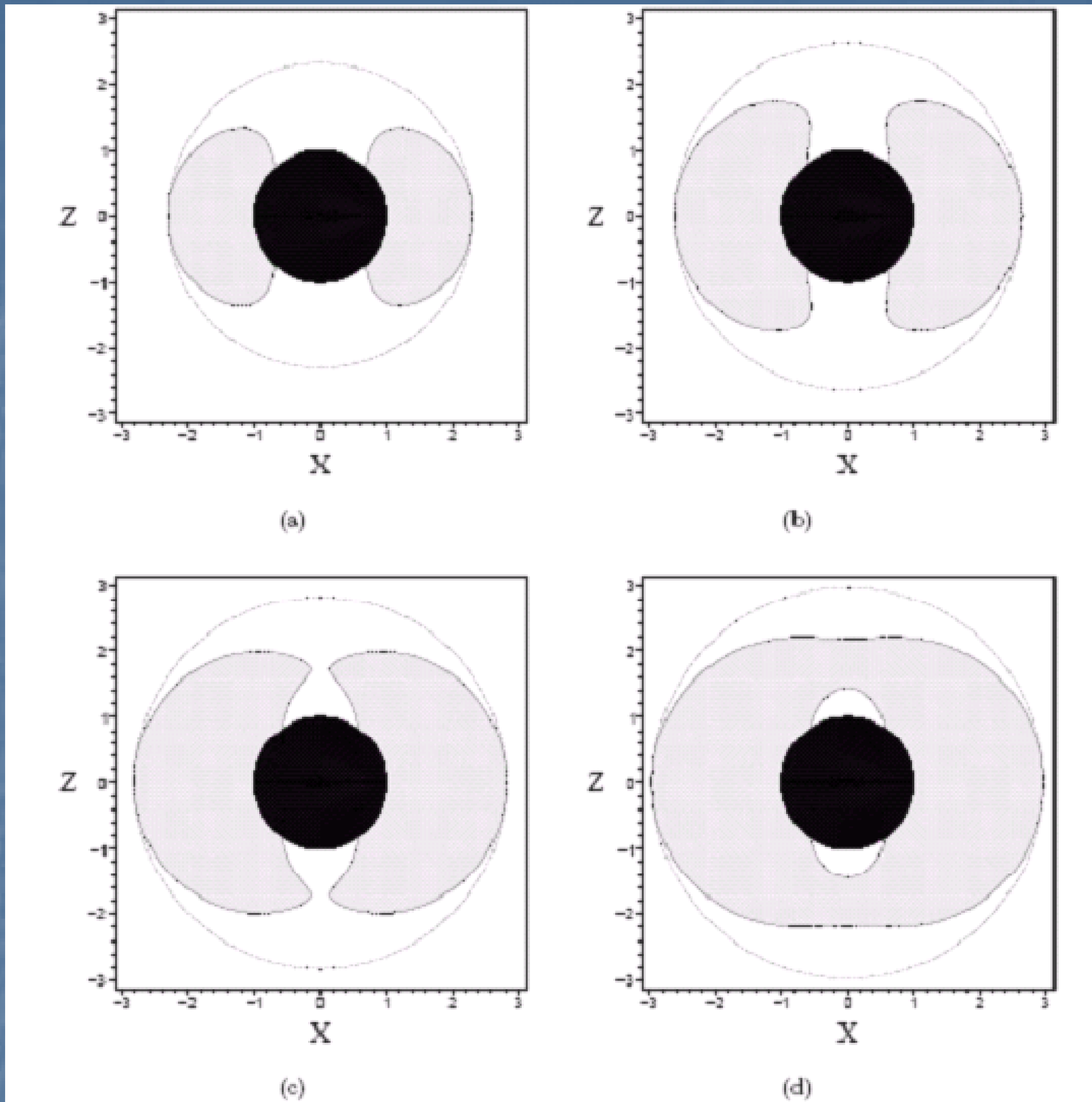
$$\left(\frac{r^d_{\pm}}{M}\right)^2 = \frac{1}{2}\frac{\xi}{\mu\epsilon} - \alpha^2\cos^2\theta \pm \left[\frac{1}{4}\frac{\xi^2}{\mu^2\epsilon^2} - 2\frac{\xi}{\mu\epsilon}\alpha^2\cos^2\theta\right]^{1/2} ,$$

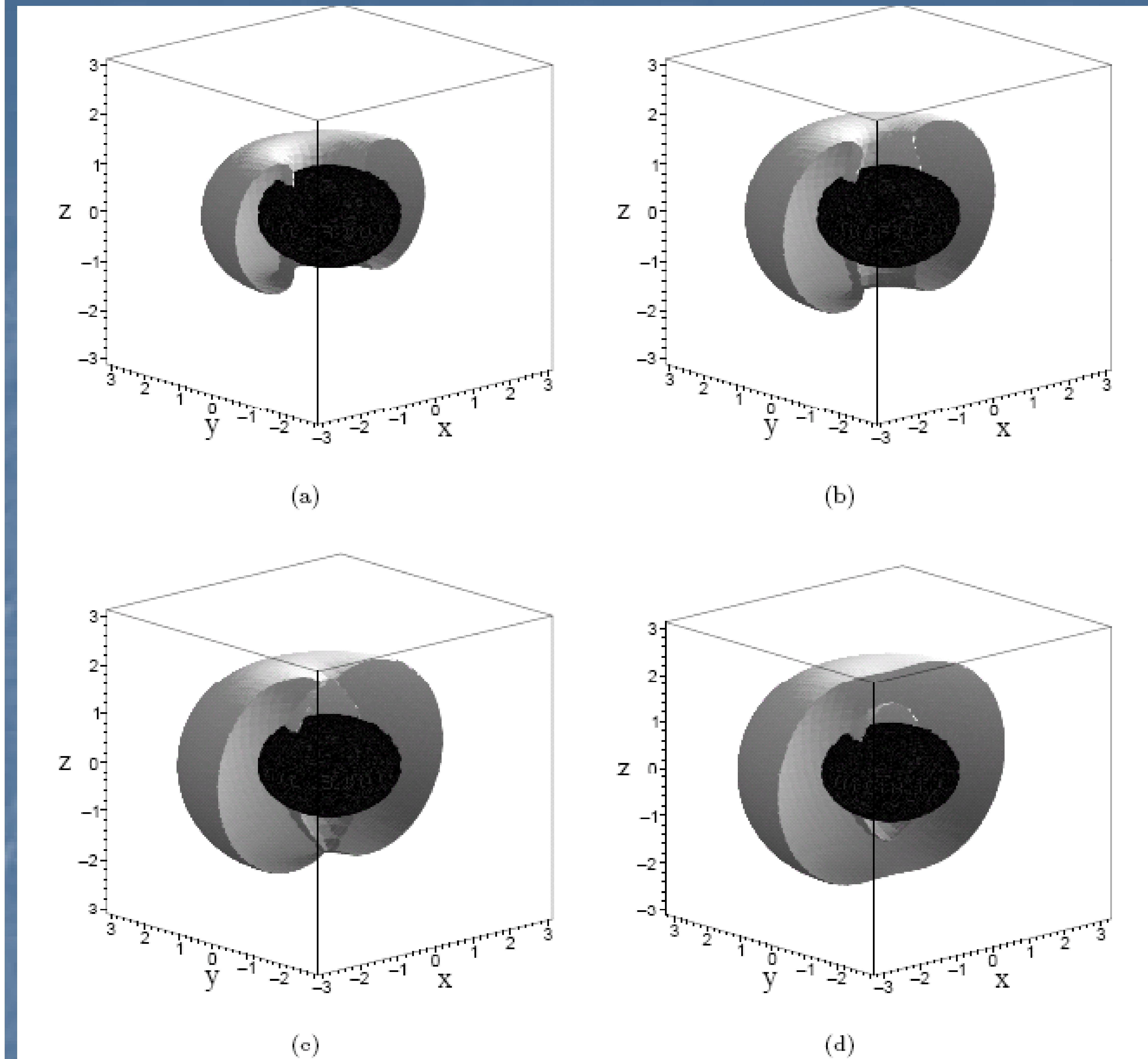
This dimensionless surface has different shapes associated with parameters…

Let's introduce Kerr-Schild coordinates (quasi-cartesian)

$$d\tilde{t} = dt - \frac{2Mr - Q^2}{\Delta}dr\ , \qquad d\psi = d\phi - \frac{a}{r^2+a^2}\frac{2Mr - Q^2}{\Delta}dr\ ,$$

$$x = \sqrt{r^2+a^2}\sin\theta\cos\psi\ , \qquad y = \sqrt{r^2+a^2}\sin\theta\sin\psi\ , \qquad z = r\cos\theta\ .$$





Ordinary spherical coords

Kerr-Schild coords.

FIG. 2: The projection of the "dyadotorus" on the $X-Z$ plane ($X = r\sin\theta$, $Z = r\cos\theta$ are Cartesian-like coordinates built up simply using the Boyer-Lindquist radial and angular coordinates) is shown for an extreme Kerr-Newman black hole with $\mu = 10$ and different values of the charge parameter $\xi = [1, 1.3, 1.49, 1.65] \times 10^{-4}$ (from (a) to (d) respectively). The dashed curves correspond to the case of vanishing rotation parameter, i.e. the Reissner-Norström case, with the "dyadotorus" which simply reduces to a sphere. The black circle represents the black hole horizon. Note that these are only representations, not embeddings.

$$|\mathbf{E}|/E_c = k$$

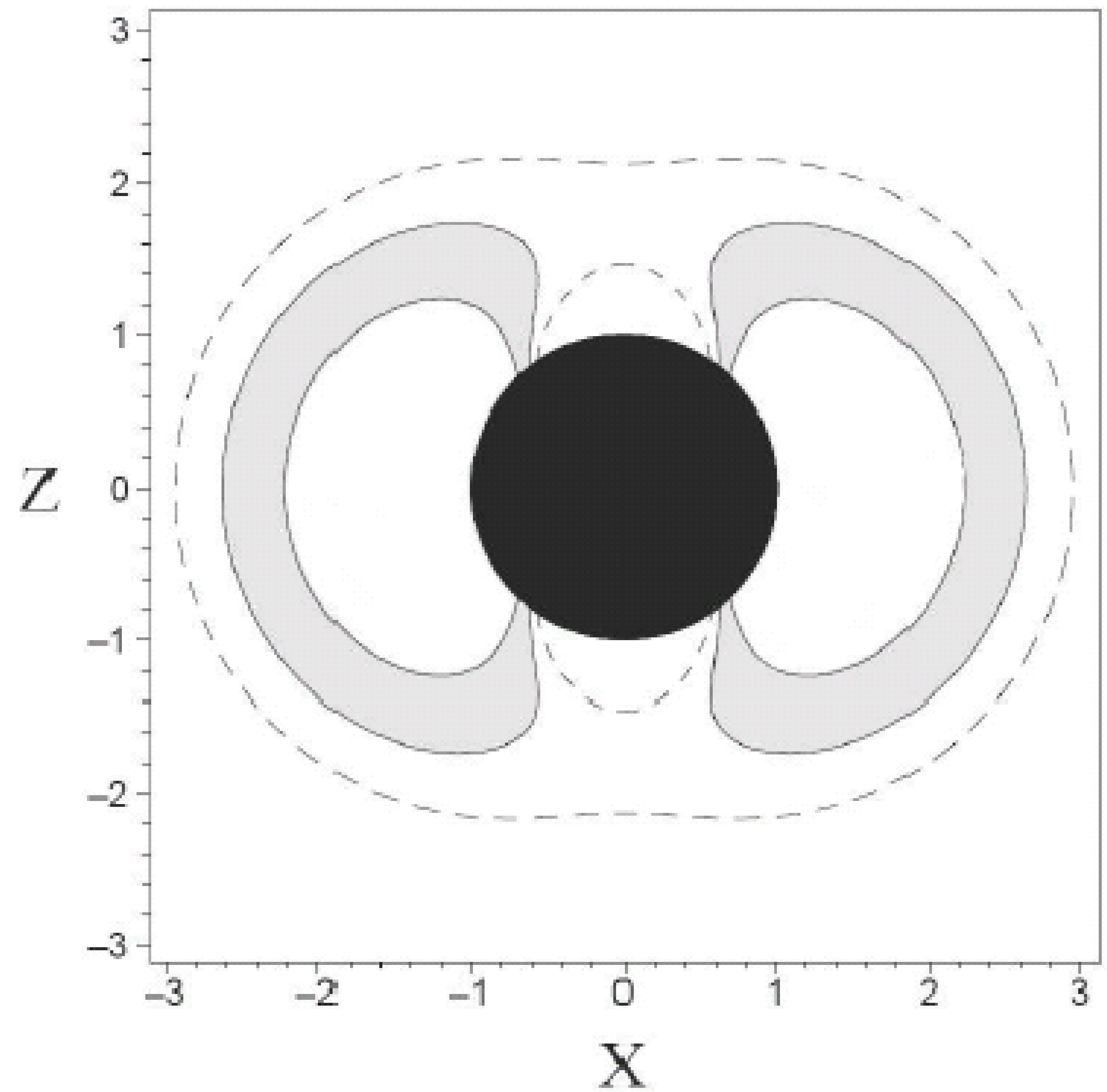


FIG. 3: The projections of the surfaces corresponding to different values of the ratio $|\mathbf{E}|/E_c \equiv k$ are shown for the same choice of parameters as in Fig. 2 (b), as an example. The gray shaded region is part of the "dyadotorus" corresponding to the case $k = 1$ as plotted in Fig. 2 (b). The region delimited by dashed curves corresponds to $k = 0.8$, i.e. to a value of the strength of the electric field smaller than the critical one, and contains the "dyadotorus;" the latter in turn contains the white region corresponding to $k = 1.4$, i.e. to a value of the strength of the electric field greater than the critical one.

When does the "torus" exist?

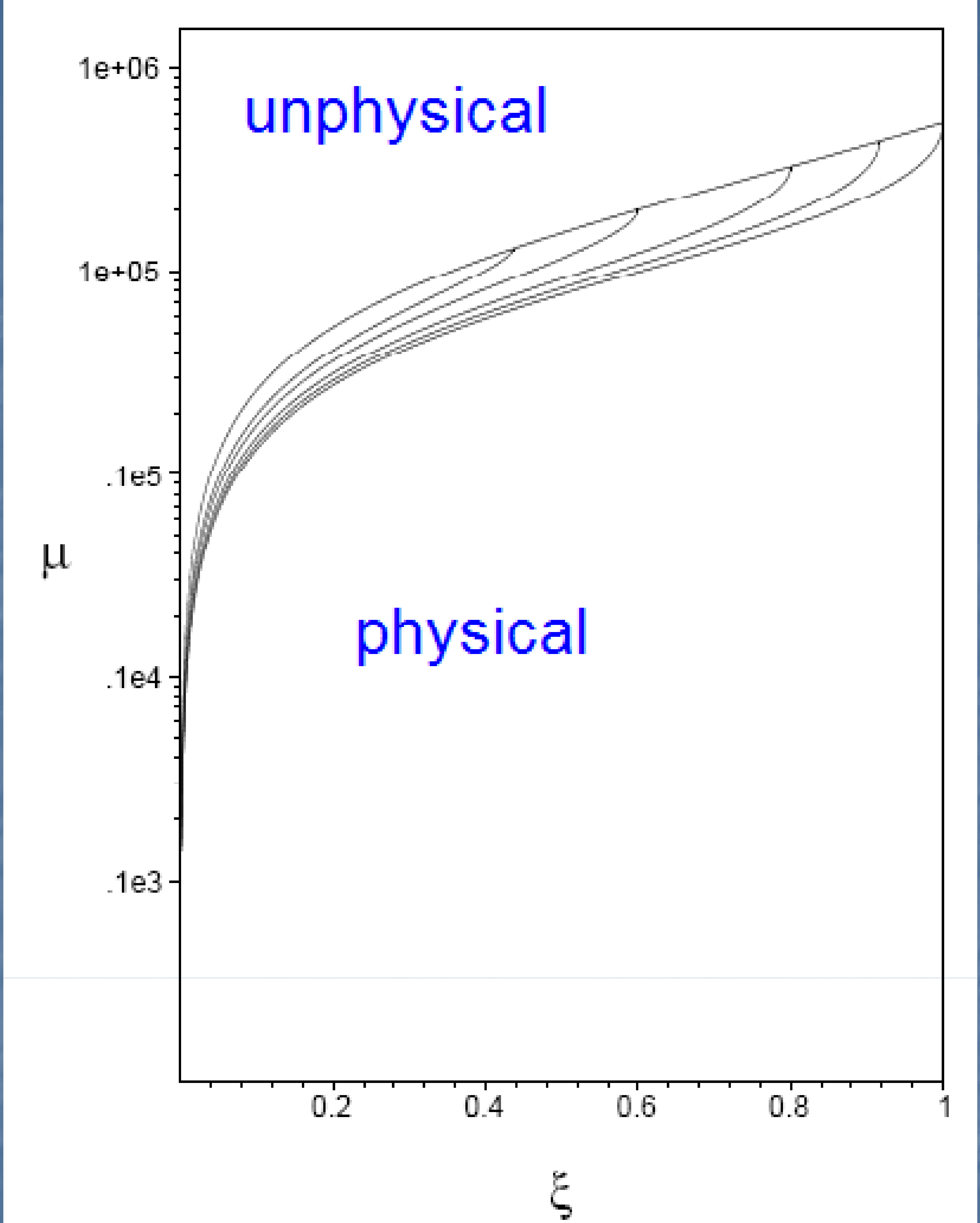
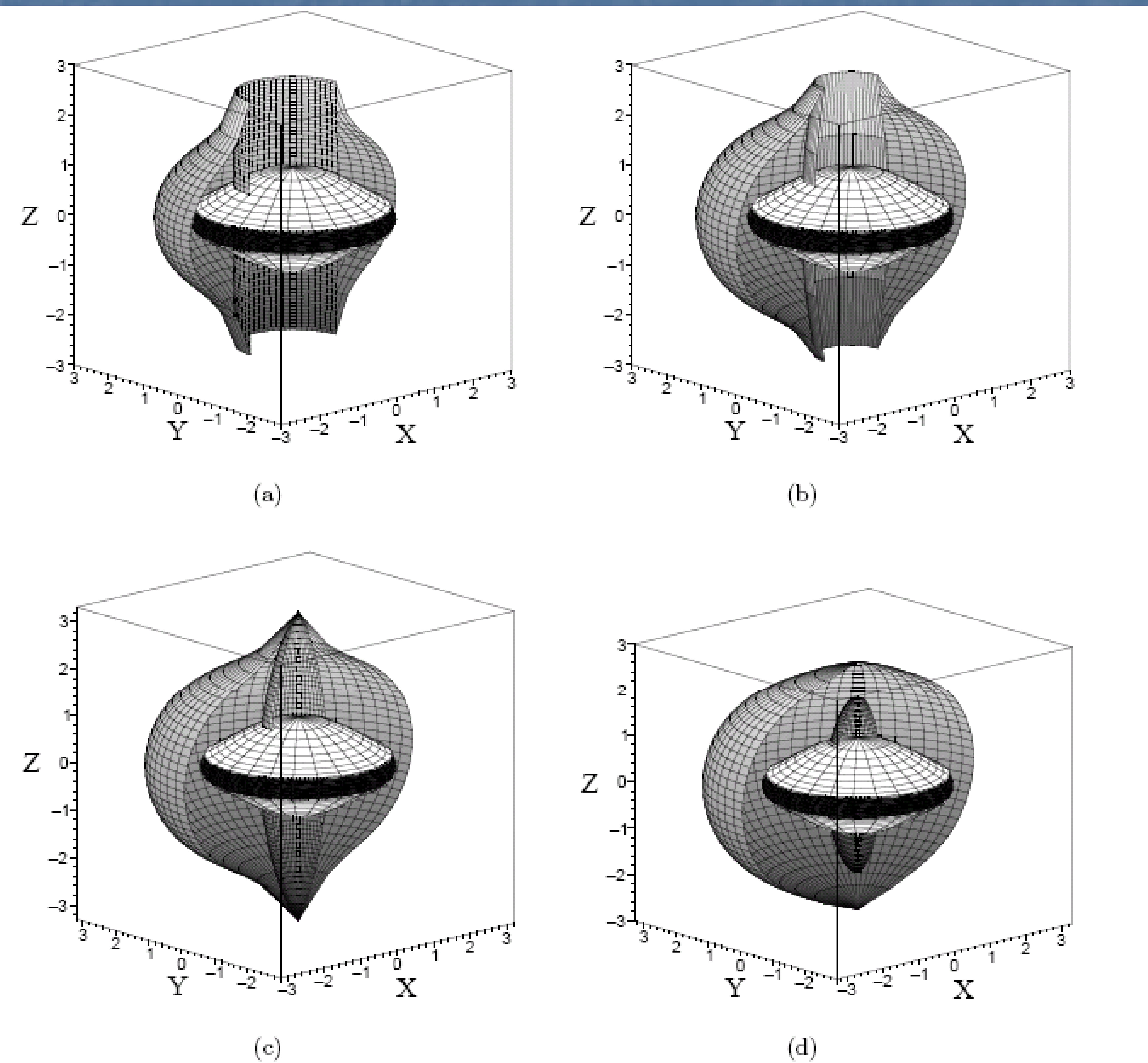
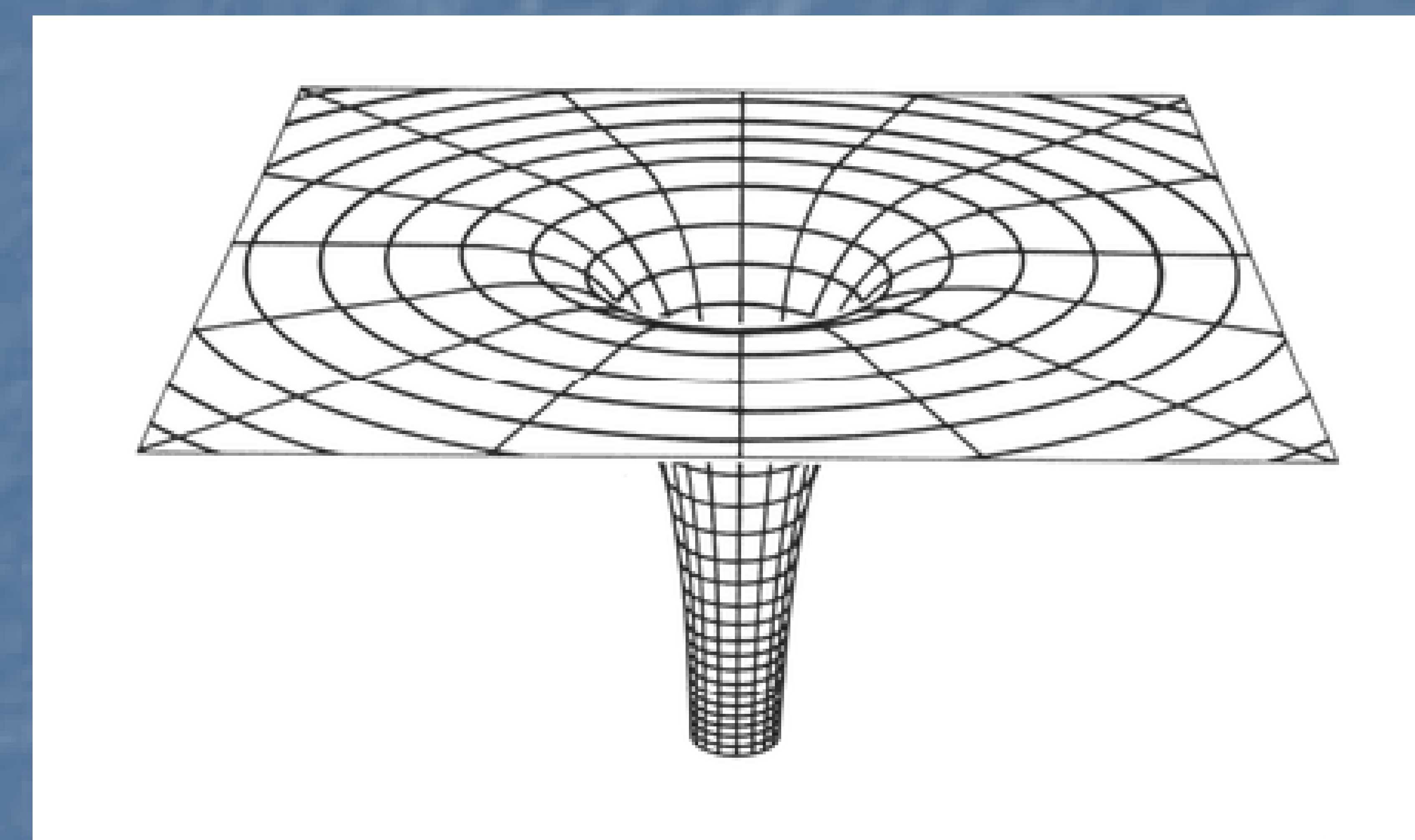


FIG. 1: The space of parameters (ξ, μ) is shown for different values of the rotation parameter $\alpha = [0, 0.4, 0.6, 0.8, 0.9, 1]$ and fixed value of the polar angle $\theta = \pi/3$ (from bottom to top). The region below each curve represents the allowed region for the existence of the "dyadotorus" with fixed α. The configurations above each line correspond to unphysical solutions where $r^q_\pm < r_+$ for the selected set of parameters.

The "dyadotorus" can be visualized as a 2-dimensional surface of revolution around the rotation axis embedded in the usual Euclidean 3-space by suppressing the temporal and azimuthal dependence.





This procedure of embedding is done in every textbooks of GR for Schwarzschild solution.

Boyer-Lindquist coordinates

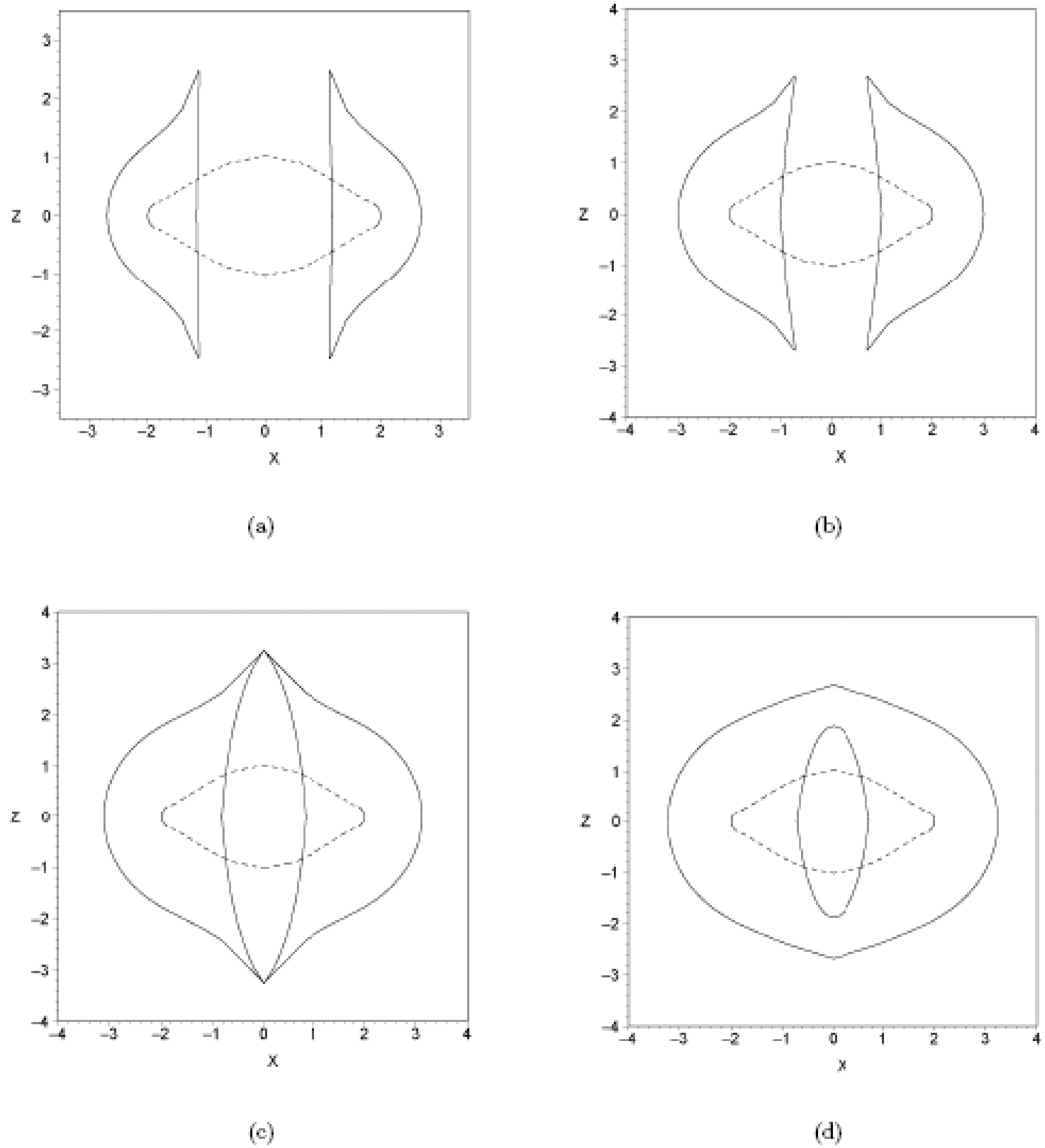


FIG. 6: The projections on the $X - Z$ plane of the embedding diagrams of Fig. 5 are shown. Dashed lines correspond to the Minkowskian part of the embedding of the outer horizon.

Here the problem of the energy inside the torus comes now, or stated more simply: what is the energy inside (??!) a finite-sized region around a KN black hole?

Let's study the Gravitational Mass of an asympthotically flat spacetime i.e. its Total Energy.

We follow the mathematics of:

Bardeen, Carter and Hawking

"The four laws of black hole mechanics"

Commun. Math. Phys. 31, p. 161 (1973)

In a stationary axisymmetric asymptotically flat space, there is a unique time translational Killing vector K^a which is timelike near infinity with $K^a K_a = -1$ and a unique rotational Killing vector $\tilde{K}^a$ whose orbits are closed curves with parameter length 2π. These Killing vectors obey equations

$$K_{a;b} = K_{[a;b]}, \qquad \tilde{K}_{a;b} = \tilde{K}_{[a;b]}, \qquad K^{a;b}{}_{b} = -R^a{}_b K^b,$$

$$K_{a;b}\tilde{K}^b = \tilde{K}_{a;b}K^b, \qquad \tilde{K}^{a;b}{}_{b} = -R^a{}_b \tilde{K}^b,$$

Crucial:it's a four velocity (a timelike observer) at spatial infinity only!

Papapetrou fields

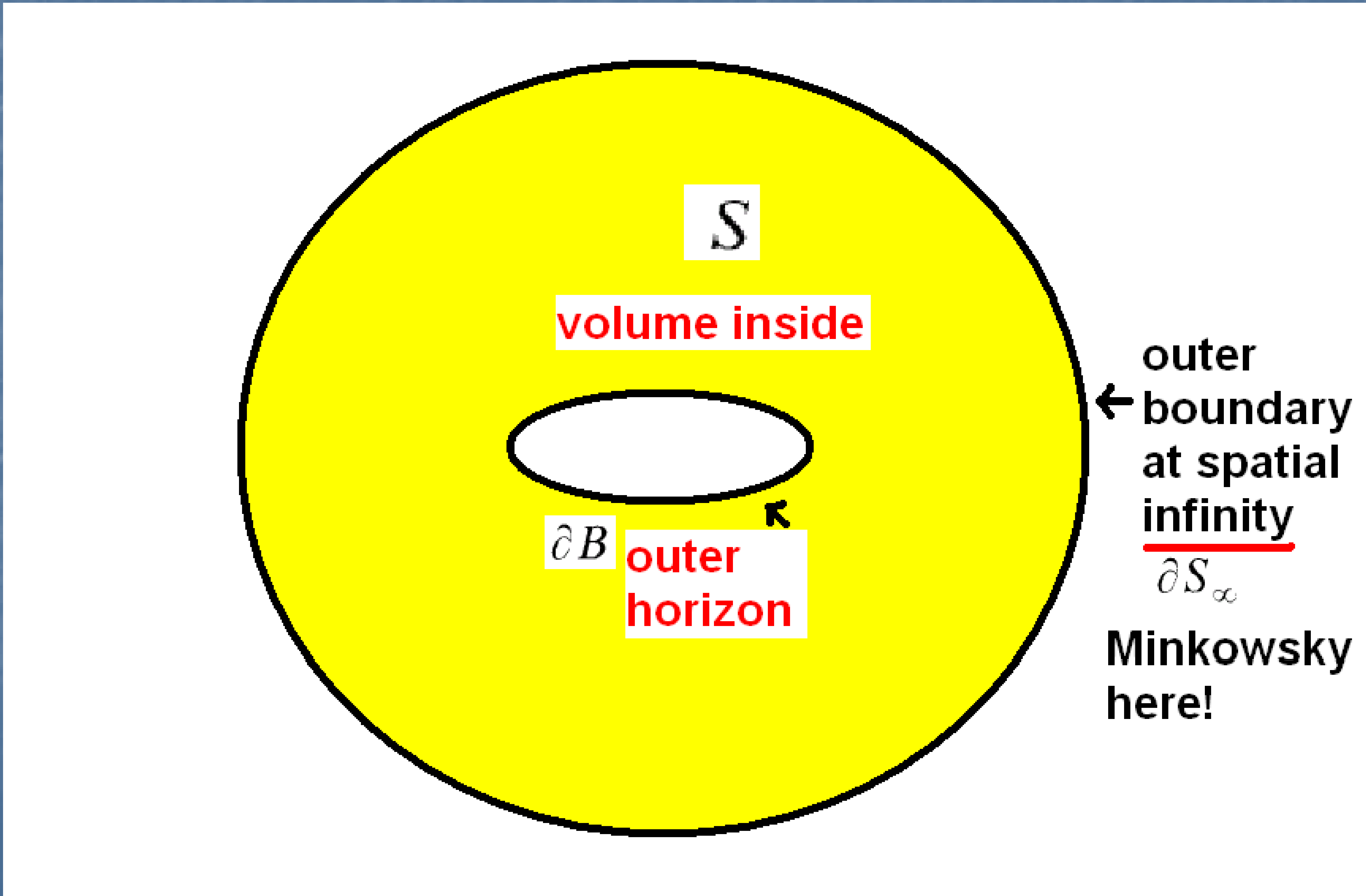
Since $K_{a;b}$ is antisymmetric, one can integrate Eq. (3) over a hypersurface S and transfer the volume on the left to an integral over a 2-surface ∂S bounding S:

(Komar Integral)

$$\int_{\partial S} K^{a;b} d\Sigma_{ab} = -\int_S R^a_b K^b d\Sigma_a, \tag{5}$$

where $d\Sigma_{ab}$ and $d\Sigma_a$ are the surface elements of ∂S and S respectively.

The domain under exam extends to infinity (to flat spacetime)!



We shall choose the surface to be spacelike, asymptotically flat, tangent to the rotation Killing vector $\tilde{K}^a$, and to intersect the event horizon [1] in a 2-surface ∂B. The boundary ∂S of S consists of ∂B and a 2-surface ∂S_∞ at infinity. For an asymptotically flat space, the integral over ∂S_∞ in equation (5) is equal to $-4\pi M$, where M is the mass as measured from infinity. Thus

$$M = \int_S (2T_a{}^b - T\delta_a^b) K^a d\Sigma_b + \frac{1}{4\pi} \int_{\partial B} K^{a;b} d\Sigma_{ab}\,, \tag{6}$$

where

$$R_{ab} - \tfrac{1}{2} R g_{ab} = 8\pi T_{ab}\,.$$

Mass at infinity!!!

factor 2

trace part...

Not simply Tmunu!!!

The first integral on the right can be regarded as the contribution to the total mass of the matter outside the event horizon, and the second integral may be regarded as the mass of the black hole.

The matter in this computation extends to space infinity!

Let's come back to the azimuthal Killing Vector

Eq. (4) similarly to obtain an expression for the total angular momentum J as measured asymptotically from infinity,

$$J = -\int_S T^a{}_b \tilde{K}^b d\Sigma_a - \frac{1}{8\pi} \int_{\partial B} \tilde{K}^{a;b} d\Sigma_{ab} \,. \tag{7}$$

The first integral on the right is the angular momentum of the matter, and the second integral can be regarded as the angular momentum of the black hole.

Let's combine these two Komar Integrals

One can introduce a time coordinate t which measures the parameter distance from S along the integral curves of K^a (i.e. $t_{;a}K^a = 1$). The null vector $l^a = dx^a/dt$, tangent to the generators of the horizon, can be expressed as

$$l^a = K^a + \Omega_H \tilde{K}^a . \quad (8)$$

The coefficient Ω_H is the angular velocity of the black hole and is the same at all points of the horizon [9]. Thus one can rewrite Eq. (6) as

$$M = \int_S (2T_a{}^b - T\delta_a^b)\, K^a d\Sigma_b + 2\Omega_H J_H + \frac{1}{4\pi} \int_{\partial B} l^{a;b} d\Sigma_{ab} , \quad (9)$$

where

$$J_H = -\frac{1}{8\pi} \int_{\partial B} \tilde{K}^{a;b} d\Sigma_{ab}$$

is the angular momentum of the black hole. One can express $d\Sigma_{ab}$ as $l_{[a}n_{b]}dA$, where n_a is the other null vector orthogonal to ∂B, normalized so that $n_a l^a = -1$, and dA is the surface area element of ∂B. Thus the last term on the right of Eq. (9) is

$$\frac{1}{4\pi} \int_{\partial B} \kappa \, dA ,$$

where $\kappa = -l_{a;b} n^a l^b$ represents the extent to which the time coordinate t is not an affine parameter along the generators of the horizon. One can think of κ as the "surface gravity" of the black hole in the following sense:

Final result for the integral mass formula

$$M = \int_S \left(2T_a^{\ b} - T\delta_a^b\right) K^a d\Sigma_b + 2\Omega_H J_H + \frac{\kappa}{4\pi} A ,$$

Remember, it was defined as the mass at infinity...

where A is the area of a 2-dimensional cross section of the horizon. When T_{ab} is zero, i.e. when the space outside the horizon is empty, this formula reduces to that found by Smarr [7] for the Kerr solution. In the Kerr solution,

BH ang. Veloc.

$$\Omega_H = \frac{J_H}{2M(M^2 + (M^4 - J_H^2)^{1/2})} , \tag{14}$$

Surface grav.

$$\kappa = \frac{(M^4 - J_H^2)^{1/2}}{2M(M^2 + (M^4 - J_H^2)^{1/2})} , \tag{15}$$

Hor. area

$$A = 8\pi(M^2 + (M^4 - J_H^2)^{1/2}) . \tag{16}$$

Not very transparent physically but one can rearrange terms…

Note: in Kerr E-M tensor is zero, i.e.the surface must not be taken necessarly at infinity…and Mass is a boundary term (holographic term)

For a Kerr-Newman Black hole

$$\Omega_H = \frac{a}{r_+^2 + a^2} ,$$
$$\kappa = \frac{r_+ - M}{r_+^2 + a^2} ,$$
$$J_H = \frac{a}{2r_+}(r_+^2 + a^2)\left\{1 + \frac{Q^2}{2a^2}\left[1 - \frac{r_+^2 + a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]\right\} ,$$
$$E_{em}^{\rm KN} = \frac{1}{4}\frac{Q^2}{ar_+^2}(r_+^2 + a^2)\arctan\left(\frac{a}{r_+}\right) + \frac{Q^2}{4r_+} . \quad (13)$$

It is easy to show that rearranging terms gives

$$M = 2\Omega_H L + \frac{\kappa}{4\pi}A + \Phi_H Q , \qquad \Phi_H = \frac{Qr_+}{r_+^2 + a^2} , \quad (14)$$

which coincides with Eq. (4) since $\Omega_H \equiv \Omega$, $\kappa \equiv 8\pi T$ and $\Phi_H \equiv \Phi$.

Rearranging terms one can obtain more transparently:

The mass formula for a charged rotating black hole is due to Christodoulou and Ruffini

$$M^2 = \left[M_{\rm ir} + \frac{Q^2}{4M_{\rm ir}}\right]^2 + \frac{L^2}{4M_{\rm ir}^2} , \quad (1)$$

where M, Q and $L = aM$ are the mass, charge and angular momentum of the black hole, and $M_{\rm ir}$ is the so called irreducible mass, related to the surface area of the horizon by

$$A - 16\pi M_{\rm ir}^2 . \quad (2)$$

$$M = \int_S (2T_a{}^b - T\delta_a^b)\, K^a d\Sigma_b + 2\Omega_H J_H + \frac{\kappa}{4\pi} A\,,$$

interpreted as a mass at infinity using the linear theory

This term is related in some sense to the mechanical energy and in this formula is meaningful at infinity only, where we can associate it to the mass.

Remember that K is timelike and normalized at spatial infinity only...

What can we do if we are interested in having the Mass (or the energy) inside a finite sized portion of spacetime...? For example inside the dyadotorus...seen by a local observer or from infinity...

The Problem now is to

localize energy in a finite-sized region

including the black hole...(important for BH thermodynamics, or for numerical Relativity,...)

The definition of M(R) is a problem (except in spherical simmetry where we have the Misner-Sharp mass). We are trying to localize energy in GR, but we know from the Equivalence Principle that this is a unnatural operation (we can remove local effect of gravity going in a free falling frame)...

We need “Quasilocal Quantities”.

Quasilocal (TOTAL) energy is an open problem. Many results are valid in specific coordinates systems, which clearly is an unnatural result 4 GR. There are plenty definitons of Mass:

Bondi, Komar, Penrose, ADM, Hayward, Hawking, Brown-York, MTW, Misner-Sharp,->∞

No one is completely satisfactory… see:

Quasi-Local Energy-Momentum and Angular Momentum in GR: A Review Article

László B. Szabados

On Living Reviews

We'll be interested in mechanical energy only... we follow Katz, Lynden-Bell and Bicak "Gravitational energy in stationary spacetimes" Class Quant grav 23, p.7111 (2006)

T^{μ}_{ν} were the Maxwell stress energy–momentum tensor

In any stationary spacetime we may use the timelike Killing vector ξ^{μ} to define 'static' observers whose velocities $w^{\mu} = \xi^{\mu}/\xi$ (ξ is the magnitude of ξ^{μ}) are unit vectors along ξ^{μ}.

The total mechanical energy on any cut through spacetime

$$E_M = \int_{\Sigma} T^{\mu}_{\nu} w^{\nu} \sqrt{-g}\, \mathrm{d}\Sigma_{\mu}.$$

From Gauss' Theorem this result is independent from the cut iff

$$D_{\mu}\left(T^{\mu}_{\nu} w^{\nu}\right) = 0$$

4 a "generic" cut, 1 gets a "conserved quantity" (a Charge)

Following [25] the total gravitational energy is

$$E_G = Mc^2 - E_M ,$$

where Mc^2 is the total energy.

This is what Ruffini and Vitagliano did in their articles.

There are various cases in which the divergence is zero (observers "made" with Killing vectors).

<u>In Kerr and Kerr Newman in Boyer-Lindquist coords, taking t=const slices as the cut, as an example.</u>

1. **<u>if ξ is the not normalized Killing vector ("no observer").</u>**

2. **<u>if ξ is the normalized Killing vector (Static observer, non geodetic)</u>**

3. **<u>if ξ is ZERO angular momentum obs. (ZAMO, non geodetic)</u>**

All these observers have problems at certain distance. Static dies at ergosphere, while ZAMO at horizon. Both give infinite energy somewhere...

$$
\begin{aligned}
E^{\mathrm{KN}}_{dya} &= \int_{r_+}^{R} T^{(em)}_{\mu\nu}\xi^\mu d\Sigma^\nu \\
&= \frac{1}{2}(M - I^H_S) - \int_R^\infty T^{(em)}_{\mu\nu}\xi^\mu d\Sigma^\nu \\
&= \frac{Q^2}{4r_+} - \frac{Q^2}{4R} + \frac{1}{4}\frac{Q^2}{ar_+^2}(r_+^2 + a^2)\arctan\left(\frac{a}{r_+}\right) \\
&\quad -\frac{1}{4}\frac{Q^2}{aR^2}(R^2 + a^2)\arctan\left(\frac{a}{R}\right) \ . \qquad (15)
\end{aligned}
$$

$\xi = \partial_t$ is the timelike Killing vector

In the limit of vanishing rotation parameter the previous equation becomes

$$
E^{\mathrm{RN}}_{dya} = \frac{Q^2}{2r_+}\left(1 - \frac{r_+}{R}\right) \ . \qquad (16)
$$

t=const cut. Finite Result,

but “not an observer”...

We need an observer which arrives all way down to the horizon, because the dyadotorus intersects both horizon and ergosphere.

Let's use geodetic observers in Doran-Painleve'-Gullstrand like horizon penetrating coordinates

Finally, the Kerr-Newman metric in the Painlevé-Gullstrand coordinates is given by

$$\begin{aligned} ds^2 \;=\; & -\left(1-\frac{2Mr-Q^2}{\Sigma}\right)dT^2+2\sqrt{\frac{2Mr-Q^2}{r^2+a^2}}dTdr-\frac{2a(2Mr-Q^2)}{\Sigma}\sin^2\theta dTd\Phi \\ & +\sin^2\theta\left[r^2+a^2+\frac{a^2(2Mr-Q^2)}{\Sigma}\sin^2\theta\right]d\Phi^2-2a\sin^2\theta\sqrt{\frac{2Mr-Q^2}{r^2+a^2}}drd\Phi \\ & +\frac{\Sigma}{r^2+a^2}dr^2+\Sigma d\theta^2 \;, \end{aligned}$$

with associated electromagnetic field

$$\begin{aligned} F \;=\; & \frac{Q}{\Sigma^2}(r^2-a^2\cos^2\theta)dr\wedge[dT-a\sin^2\theta d\Phi] \\ & +2\frac{Q}{\Sigma^2}ar\sin\theta\cos\theta d\theta\wedge[(r^2+a^2)d\Phi-adT] \;, \end{aligned}$$

which has the same form as (3) with $dt\to dT$ and $d\phi\to d\Phi$.

What are these coordinates? Let's see the simple a=0 case.

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dT^2 + 2\frac{\sqrt{2Mr - Q^2}}{r} dT\,dr$$

$$+ dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \; ,$$

flat 3-geometry

$$F = \frac{Q}{r^2} dr \wedge dT \; .$$

Shift dies at r=Q^2/2M. Here gravity becomes repulsive, but it's unphysical because this region is inside the inner horizon which is a Cauchy horizon (unphysical)

T is the proper time as measured by a free falling observer starting from rest at infinity and and moving radially inward

Regular Metric, constant Lapse, No stretched horizon anymore!!! Very useful in numerical relativity. PG naturally appears in acoustic black holes…deep thing!

slicing observers (T–slicing hereafter) have 4-velocity

$$\mathcal{N} = \partial_T - \frac{\sqrt{(2Mr - Q^2)(r^2 + a^2)}}{\Sigma}\partial_R \ .$$

Energy density in KN

$$\mathcal{E}(\mathcal{N}) = T^{(\mathrm{em})}_{\mu\nu}\mathcal{N}^\mu\mathcal{N}^\nu = \frac{Q^2}{8\pi\Sigma^3}(r^2 - a^2\cos^2\theta + 2a^2) \ ,$$

Quasilocal mechanical energy (T=const cut)

$$E(\mathcal{N})_{(r_+,R)} \ = \ 2\pi\int_{r_+}^{R}\int_0^\pi \mathcal{E}(\mathcal{N})\sqrt{h}\,dr\,d\theta = -\frac{Q^2}{4a}\left[\frac{a}{r} + \frac{r^2+a^2}{r^2}\arctan\frac{a}{r} - \frac{\pi}{2}\right]_{r_+}^{R}$$

$T \ = \ const$ hypersurface

To avoid integration troubles we appoximate the integration domain with a r= constant ellipsoid which contains the ergoregion

Pain. Gull.

$$T^{(\mathrm{em})}_{\mu\nu}\mathcal{N}^{\mu}$$

is not divergence free!

no conserved charge

But our T=constant cut for this PG machinery gives:

$$E(\mathcal{N})_{(r_+,R)} = 2\pi\int_{r_+}^{R}\int_0^{\pi}\mathcal{E}(\mathcal{N})\sqrt{h}\,dr\,d\theta = -\frac{Q^2}{4a}\left[\frac{a}{r}+\frac{r^2+a^2}{r^2}\arctan\frac{a}{r}-\frac{\pi}{2}\right]_{r_+}^{R}$$

Which COINCIDES WITH

$$E^{\mathrm{KN}}_{dya} = \int_{r_+}^{R} T^{(em)}_{\mu\nu}\xi^{\mu}d\Sigma^{\nu}$$

$\xi = \partial_t$ is the timelike Killing vector

(Boy.Lindq)

which appears in the mass formula and in Ruffini-Vitagliano's work.

Summarizing

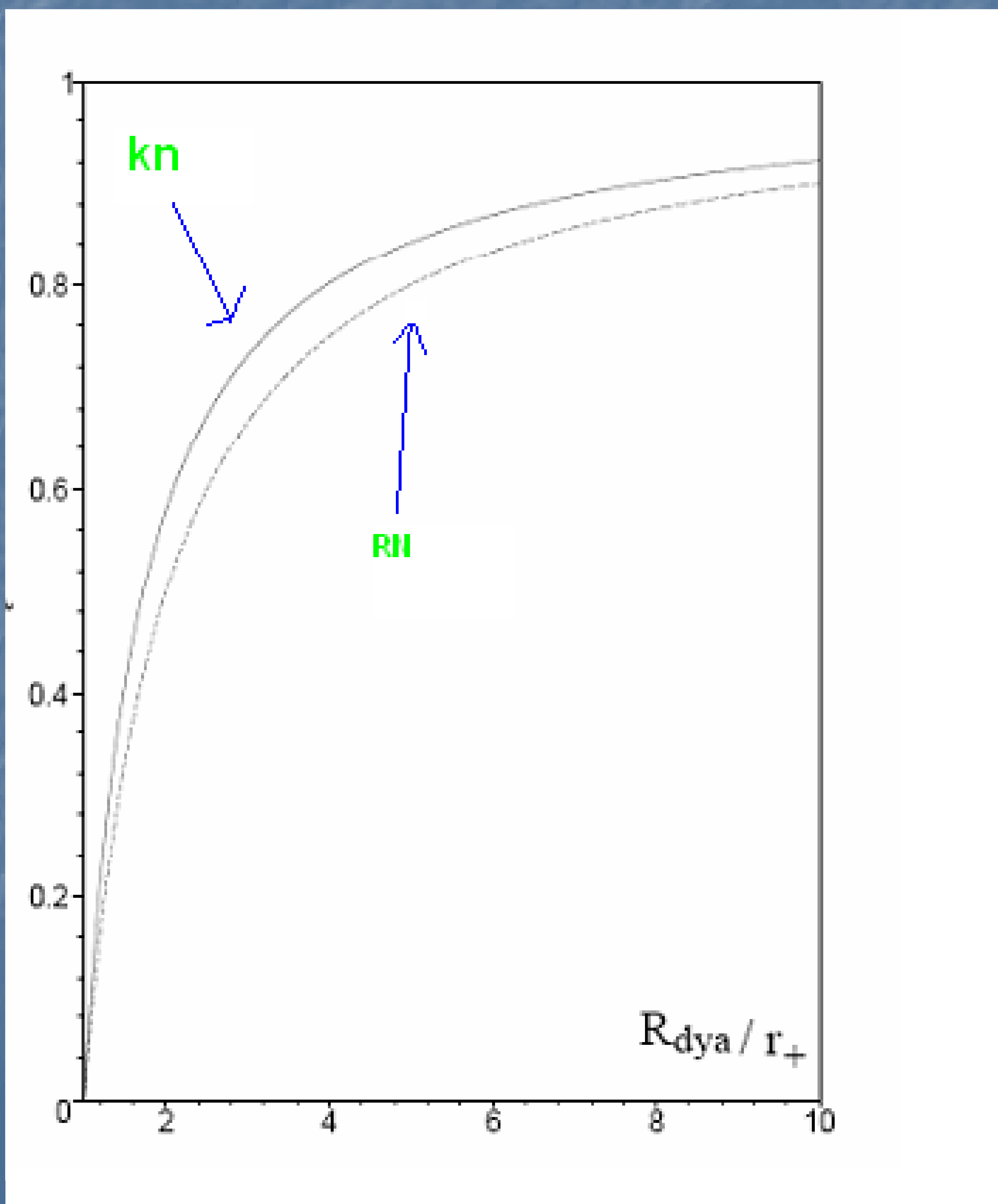
- IN BOYER-LINDQUIST COORDS, THE NON NORMALIZED TIME KILLING VECTOR (Vitagliano-Ruffini) GIVES A FINITE CONSERVED QUANTITY, BUT THIS "MECHANICAL ENERGY" IS MEANINGFUL AT SPATIAL INFINITY ONLY, **IT'S NOT A QUASILOCAL CONCEPT**.
- IN BOYER-LINDQUIST COORDS, NON GEODESIC OBSERVERS GIVES A CONSERVED QUANTITY WHICH HOWEVER DIVERGES SOMEWHERE CLOSE TO THE BH: **IT'S A QUASILOCAL ENERGY BUT NOT GLOBALLY WELL DEFINED**.
- IN HORIZON PENETRATING COORDINATES (P-G) A GEODESIC OBSERVER (with T=const cuts) GETS A QUASILOCAL ENERGY (WELL DEFINED EVERYWHERE) WHICH **IS NOT A CONSERVED CHARGE** BUT AT INFINITY GIVES THE MASS FORMULA RESULT.

We consider the energy measured by this observer as an acceptable quasilocal energy which gives the correct whole spacetime limit! This result is the same as Ruffini-Vitagliano's one

We can consequently define for this observer:

$$\lambda = \frac{E(\xi)_{(r_+,R_{\rm dya})}}{E(\xi)_{(r_+,\infty)}} = 1 - \left(\frac{r_+}{R_{\rm dya}}\right)^2 \left[\frac{aR_{\rm dya} + (R_{\rm dya}^2 + a^2)\arctan(a/R_{\rm dya})}{ar_+ + (r_+^2 + a^2)\arctan(a/r_+)}\right]$$

$$\lambda^{\rm RN} = 1 - \frac{r_+}{R_{\rm dva}}$$



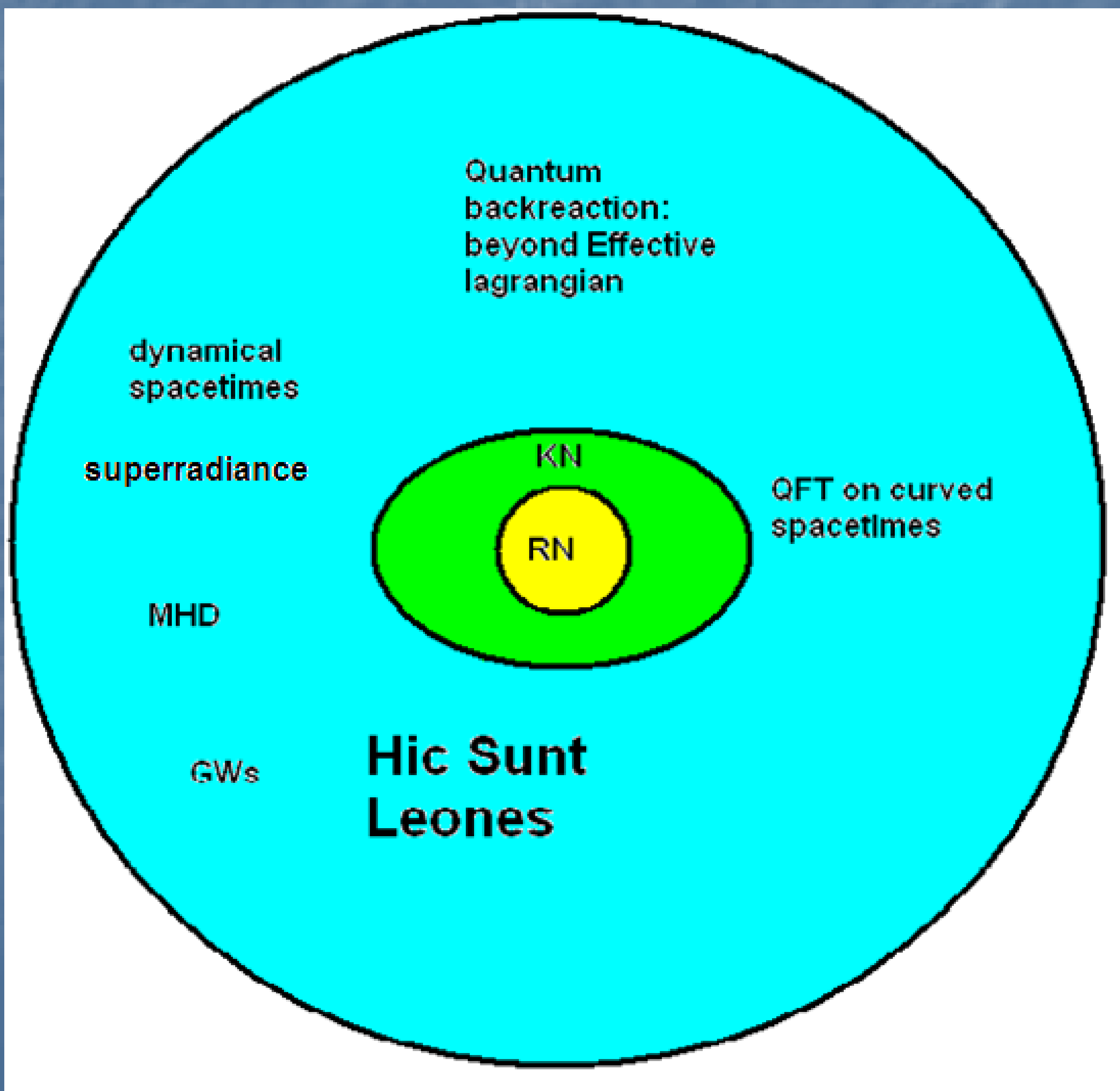
Extreme highly rotating Kerr-Newman Black hole and (non extreme) Reissner-Nordstrom both with

xi=Q/ M =1.3 x 10E-4 and 10 solar masses.

Notice: KN is more compact because horizon is smaller than RN (around a half!).

Concluding Remarks

Kerr-Newman black hole can be a very compact source for possible QED process "a la Damour Ruffini" but with respect to Reissner-Nordstrom we'll have much more complications:



The Dyado Atlas

MHD (particles moving in rotating EM fields)

GWS (non spherical pair production)

BACKREACTION

AND DYNAMICAL SPACETIMES

NO KERR-NEWMAN PERTURBATION THEORY!!!

Here be lions, really!!!