GWs from neutron star oscillations: comparisons between linear and nonlinear evolutions

Pescara, July 18th 2008
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Outline of the work

GWs from even-parity oscillation of a perturbed TOV star

Compare the results obtained from
3D FGR simulations with perturbative ones (1D, linear)

✓ Zerilli extraction
✓ $\Psi_4$ extraction
✓ Quadrupole formulas
✓ Non-linear effects (as a function of the amplitude of the initial perturbation)
Motivation

GWs from NS oscillations

- excited e.g. after Supernova Core Collapse
- non-linear oscillations!

- test-bed for 3D wave extraction methods
  (in non-vacuum spacetimes) and for analysis methods

Why a linear time-domain code?

- Perturbative methods: quasi equilibrium systems
- 1D: computationally less expensive than 3D
- Accurate results (more resolution)

- Check 3D extraction methods
- Basis for non-linear analysis
Strategy: the double approach

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \]

\[ \delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu} \]

Cactus-Carpet-Whisky

PerBaCCo
Strategy: the double approach

\[ G_{\mu \nu} = 8 \pi T_{\mu \nu} \]

\[ \delta G_{\mu \nu} = 8 \pi \delta T_{\mu \nu} \]
1D time-domain code: **PerBaCCo**

**PerturBative Constrained Code**

- All kind of TOV perturbations (RW gauge, spherical coord.)
- Radial, Axial and Polar perturbations: (constrained) Wave Eqs
- Standard 11th order FD schemes
  - Even-parity: constrained algorithm
- Use tabulated equations of state (EOS) for nuclear matter
- Zerilli-Moncrief (even-parity) and Regge-Wheeler (odd-parity) gauge invariant functions

\[
\begin{align*}
    h_+ - ih_\times &= \frac{1}{r} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} N_l \left( \Psi^{(e)}_{l,m} + i \Psi^{(o)}_{l,m} \right) - 2 Y_{l,m}(\theta, \phi)
\end{align*}
\]

Cactus-Carpet-Whisky: setup

- Metric/Matter evolution:
  - ✓ (ADM) NOK-BSSN + GRHD Cons Form
  - ✓ gauge: “l+log” + Gamma Driver
  - ✓ MoL: ICN
  - ✓ HRSC: Marquina + PPM
- Grid:
  - ✓ 3 cubic boxes, Dx=0.5
  - ✓ Octant Sym
  - ✓ CFL = 0.25

Developed mainly @ AEI, LSU
Computer Cluster in Parma

**ALBERT**

- 16 nodes: bi-processor opteron 2 GHz
- 4 GB RAM
- 3 TB RAID 5 storage
- Infiniband

**ALBERT100**

- 32 nodes: bi-processor Pentium III - 1.5 GB RAM
- 100BaseT fast ethernet
- Peak: 100 Gflops
Initial Data: Whisky_PerturbTOV

- **TOV eqs** (Whisky_TOVSolverC)
- **Perturbation** (Whisky_PerturbTOV):
  - ✓ add pressure perturbation
  - ✓ solve (perturbative) constraints for each multipoles
  - ✓ construct perturbed metric
  -   ‣ Fix a specific multipole (1 constraint eq)
  -   ‣ Axisymmetric pressure perturbation
  -   ‣ Metric perturbation:

\[
\delta s^2_{\ell_0} = (\chi_{\ell_0} + k_{\ell_0}) e^{2a} dt^2 - 2\psi_{\ell_0} e^{a+b} dt d\bar{r} \\
+ e^{2b} \left[ (\chi_{\ell_0} + k_{\ell_0}) d\bar{r}^2 + \bar{r}^2 k_{\ell_0} d\Omega \right] Y_{\ell_0}
\]
Matter perturbation

Perturbed pressure:

\[ \delta p(r, \theta) \equiv (p + \mu) H_{\ell 0}(r) Y_{\ell 0}(\theta) \]

Enthalpy profile:

\[ H = h \sin \left[ \frac{(n + 1) \pi r}{2R} \right] \]

Quadrupolar mode:

\[ \ell = 2 \]
\( (\text{Linearised}) \text{ Hamiltonian constraint solution} \)

\[
\begin{align*}
&\left(1 - \frac{2m}{r}\right) k_{,rr} + \left[\frac{2}{r} - \frac{3m}{r^2} - 4\pi\varepsilon r\right] k_{,r} - \left[\frac{\Lambda}{r^2} - 8\pi\varepsilon\right] k = \\
&- \frac{8\pi(p + \varepsilon)}{C_s^2} H + \left(1 - \frac{2m}{r}\right) \chi_{,r} + \left[\frac{2}{r} - \frac{2m}{r^2} + \frac{\Lambda}{2r} - 8\pi\varepsilon r\right] \chi
\end{align*}
\]

Conformally Flat 1D (fluid modes):

\( \chi_{\ell 0} = 0 \)
Equilibrium model and radial modes

- **Perfect fluid, Polytropic Model A0**  \( M = 1.4M_\odot \rho_c = 1.28 \times 10^{-3} \)  \( R = 9.57 \)

- **Stable Evolution unperturbed model (Radial Modes)**

<table>
<thead>
<tr>
<th>n</th>
<th>Pert. [Hz]</th>
<th>3D [Hz]</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1462</td>
<td>1466</td>
<td>0.3</td>
</tr>
<tr>
<td>1</td>
<td>3938</td>
<td>3935</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>5928</td>
<td>5978</td>
<td>0.8</td>
</tr>
</tbody>
</table>

- **Stable Evolution of the sequence AU**
  (Uniformly rotating models and fixed mass)

<table>
<thead>
<tr>
<th>MODEL</th>
<th>F [Hz]</th>
<th>F(CF) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU0</td>
<td>1466</td>
<td>1458</td>
</tr>
<tr>
<td>AU1</td>
<td>1369</td>
<td>1398</td>
</tr>
<tr>
<td>AU2</td>
<td>1329</td>
<td>1345</td>
</tr>
<tr>
<td>AU3</td>
<td>1265</td>
<td>1283</td>
</tr>
<tr>
<td>AU4</td>
<td>1166</td>
<td>1196</td>
</tr>
<tr>
<td>AU5</td>
<td>1093</td>
<td>1107</td>
</tr>
</tbody>
</table>

[Dimmelmeier et al 2007]
Even-parity perturbative waves: identikit

1. Fourier analysis
2. Fit analysis
3. Finite extraction effects

- Radial grid with 300pts inside the star
- Long evolution (about 1 sec)
1. Fourier analysis:

\[ \nu_f \quad 1581 \text{ Hz} \]
\[ \nu_{p_1} \quad 3724 \text{ Hz} \]
2. Fit analysis - QNMs template:

\[ \Psi^{(e)}_{20} \sim \sum_{k=0}^{N} A_{2k} \cos(2\pi \nu_{2k} t + \phi_{2k}) \exp(-\alpha_{2k} t) \]

\[ N = 2 \]
## 2. Fit analysis - results:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Conf-</th>
<th>Conf+</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{20}$</td>
<td>$1.5837369e+03$</td>
<td>$1.5837368e+03$</td>
<td>$1.583737e+03$</td>
</tr>
<tr>
<td>$\nu_{21}$</td>
<td>$3.7069413e+03$</td>
<td>$3.7069401e+03$</td>
<td>$3.7069424e+03$</td>
</tr>
<tr>
<td>$\alpha_{20}$</td>
<td>$3.7358$</td>
<td>$3.7349$</td>
<td>$3.7367$</td>
</tr>
<tr>
<td>$\alpha_{21}$</td>
<td>$4.22e-01$</td>
<td>$4.15e-01$</td>
<td>$4.29e-01$</td>
</tr>
<tr>
<td>$A_{20}$</td>
<td>$1.31452e-03$</td>
<td>$1.31430e-03$</td>
<td>$1.31475e-03$</td>
</tr>
<tr>
<td>$A_{21}$</td>
<td>$3.52e-05$</td>
<td>$3.50e-05$</td>
<td>$3.53e-05$</td>
</tr>
<tr>
<td>$\phi_{20}$</td>
<td>$2.809e-01$</td>
<td>$2.807e-01$</td>
<td>$2.811e-01$</td>
</tr>
<tr>
<td>$\phi_{21}$</td>
<td>$3.965e-01$</td>
<td>$3.929e-01$</td>
<td>$4.002e-01$</td>
</tr>
</tbody>
</table>

### Damping Times:

\[ \tau_f = 0.268 \text{ sec (0.1\%)} \]
\[ \tau_{p_1} = 2.28 \text{ sec (2\%)} \]
3. Finite extraction effects
3. Finite extraction effects

\[ \max \Psi^{(e)}(r) \sim A^\infty + A^1 \frac{M}{r} + \ldots \]

<table>
<thead>
<tr>
<th>r [M]</th>
<th>( \delta A/A^\infty_{fit} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>49.30%</td>
</tr>
<tr>
<td>50</td>
<td>8.90%</td>
</tr>
<tr>
<td>100</td>
<td>1.74%</td>
</tr>
<tr>
<td>200</td>
<td>1.62%</td>
</tr>
</tbody>
</table>
Comparing 1D VS 3D Waves

Different values of the initial perturbation amplitude:

Wave Extraction at $r = 80M$

$h = [0.001, 0.01, 0.05, 0.1] := [h_0, h_1, h_2, h_3]$
<table>
<thead>
<tr>
<th>$h$</th>
<th>$\nu^f_{3D}$ [Hz]</th>
<th>Diff. [%]</th>
<th>$\nu^{p1}_{3D}$ [Hz]</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>h0</td>
<td>1578</td>
<td>0.2</td>
<td>3705</td>
<td>0.5</td>
</tr>
<tr>
<td>h1</td>
<td>1576</td>
<td>0.3</td>
<td>3705</td>
<td>0.5</td>
</tr>
<tr>
<td>h2</td>
<td>1573</td>
<td>0.5</td>
<td>3635</td>
<td>2.4</td>
</tr>
<tr>
<td>h3</td>
<td>1623</td>
<td>2.7</td>
<td>3565</td>
<td>4.3</td>
</tr>
</tbody>
</table>

![Graph showing amplitude as a function of h with percentage differences highlighted]
Initial "burst"

- High frequency oscillations
- Linear grows in r
- Increases with resolution
- Greater for higher initial perturbation amplitude
- Smaller when full constraints are solved

Unphysical and related to the constraint violation and to the Zerilli 3D extraction...
Ψ₄ extraction

![Graph showing Ψ₄ extraction over time, with two lines representing 3D-code and 1D-code, and a horizontal axis labeled 'h₀ perturbation'.]
Ψ₄ extraction

Consistent with 1D
DOES NOT grows in r
How to recover the Zerilli’s?
How to recover the Zerilli’s?

\[ \Psi^{(e)}(t) \propto \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \left\{ \lim_{r \to \infty} [r \Psi^4(t'', r)] \right\} \]

\[ = Q_0 + Q_1 t + \int_{0}^{t} dt' \int_{0}^{t'} dt'' \left\{ \lim_{r \to \infty} [r \Psi^4(t'', r)] \right\} \]

\[ = Q_0 + Q_1 t + \int_{0}^{t} dt' \int_{0}^{t'} dt'' \left( r \Psi^4(t'', r) + f(t'', r) \right) \]

\[ = Q_0 + Q_1 t + \left[ \int_{0}^{t} dt' \int_{0}^{t'} dt'' r \Psi^4(t'', r) \right] + \sum_{k=2}^{n} F_k(r)t^k + \ldots \]
How to recover the Zerilli’s?

1. “off-set” function

\[ \Psi^{(e)}(t) \propto \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \left\{ \lim_{r \to \infty} \left[ r \Psi^4(t'', r) \right] \right\} \]

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\[ = Q_0 + Q_1 t + \int_{0}^{t} dt' \int_{0}^{t'} dt'' (r \Psi^4(t'', r) + f(t'', r)) \]

\[ = Q_0 + Q_1 t + \left[ \int_{0}^{t} dt' \int_{0}^{t'} dt'' r \Psi^4(t'', r) \right] + \sum_{k=2}^{n} F_k(r)t^k + \ldots \]
How to recover the Zerilli’s?

1. “off-set” function

\[ \Psi^{(e)}(t) \propto \int^{t}_{-\infty} dt' \int^{t'}_{-\infty} dt'' \left\{ \lim_{r \to \infty} [r \Psi^4(t'', r)] \right\} \]

\[ = Q_0 + Q_1 t + \int^{t}_{0} dt' \int^{t'}_{0} dt'' \left\{ \lim_{r \to \infty} [r \Psi^4(t'', r)] \right\} \]

\[ = Q_0 + Q_1 t + \int^{t}_{0} dt' \int^{t'}_{0} dt'' (r \Psi^4(t'', r) + f(t'', r)) \]

\[ = Q_0 + Q_1 t + \left[ \int^{t}_{0} dt' \int^{t'}_{0} dt'' r \Psi^4(t'', r) \right] + \sum_{k=2}^{n} F_k(r)t^k + \ldots \]

2. “slow” variation

Let’s try:

\[ \Psi^{(e)}(t, r) \propto \int^{t}_{0} dt' \int^{t'}_{0} dt'' r \Psi^4(t'', r) + Q_0 + Q_1 t + F_2 t^2 + \ldots \]
$\Psi(e)$ from 3D $\psi^4$ Corrected

$\Psi^4$ bare

floor

$u$

$\Psi(e)$

from 3D $\psi^4$ Corrected

from 3D $\Psi^4$ bare

floor
(Once corrected for the floor) Zerilli from the $\Psi_4$ extraction is perfectly consistent with the perturbative...
Non-linear effects

Mode couplings:

✓ non-axisymmetric + odd parity modes: suppressed
✓ radial modes
✓ ell=4,6 m=0,4 (grid)

STRATEGY:

Fourier analysis of “Weak” couplings

\[ \langle \rho \rangle_{\ell,m}(t) = \int d^3 x \rho(t, x) Y_{\ell,m} \]

\[ \nu_{coupl} = \nu_1 \pm \nu_2 \]
\[ \langle \rho \rangle_{2,0} \]
\[ \langle \rho \rangle_{2,0} \]
\[ \langle \rho \rangle_{2,0} \]

[Passamonti et al. 2006 / Dimmelmeier et al 2007]
**Quadrupole extraction**

**Functional form:**

\[ I_{ij}[\varrho] \equiv \int d^3x \varrho x_i x_j \]

No “Standard Quadrupole” in full GR.
Possible generalizations worth to try

**Multipole:**

\[ rh_{2,0} = \sqrt{\frac{24\pi}{5}} (\ddot{I}_{zz} - \frac{1}{3} I) \]

**SQF:** \[ \varrho = \rho \]

**SQF1:** \[ \varrho = \alpha^2 \sqrt{\gamma} T^{00} \]
[Blanchet et al 1990/ Shibata Sekiguchi 2003]

**SQF2:** \[ \varrho = \sqrt{\gamma} W \rho \]
[Nagar et al. 2005]

**SQF3:** \[ \varrho = u^0 \rho = \frac{W}{\alpha} \rho \]
[Blanchet et al 1990/ Shibata Sekiguchi 2003]

\[ h_+ - ih_\times = \sum_{\ell,m} h_{\ell,m} - 2Y_{\ell,m} \]

(S.Bernuzzi - Pescara - July, 18th 2008)
**Frequencies : OK**

**Differences in amplitude !**

\[ \text{SQF : } \varrho = \rho \]

\[ \text{SQF1: } \varrho = \alpha^2 \sqrt{\gamma T^{00}} \]

\[ \text{SQF2: } \varrho = \sqrt{\gamma W \rho} \]

\[ \text{SQF3: } \varrho = u^0 \rho = \frac{W}{\alpha} \rho \]
Frequencies : OK

Differences in amplitude !

SQF : $\varrho = \rho$

SQF1: $\varrho = \alpha^2 \sqrt{\gamma} T^{00}$

SQF2: $\varrho = \sqrt{\gamma} W \rho$

SQF3: $\varrho = u^0 \rho = \frac{W}{\alpha} \rho$
Summary

• **Perturbative ID** (*Whisky_PerturbTOV*)

• Evolve with both 3D FGR and 1D perturbative code

• **Wave extraction**: WaveExtract (Zerilli), Psikadelia (Psi4) and SQFs

• Compare results
Conclusions

• **Zerilli Extraction**
  - ✓ 3D Zerilli extraction consistent with Perturbative (linear regime)
  - ✓ Extraction r>80M
  - ✓ initial junk Zerilli Extraction

• **\( \Psi^4 \) Extraction**
  - ✓ 3D \( \Psi^4 \) extraction consistent with Perturbative (linear regime)
  - ✓ Extraction r>80M
  - ✓ NO junk radiation
  - ✓ Off-set subtraction needed
Conclusions

- **Zerilli Extraction**
  - ✓ 3D Zerilli extraction consistent with Perturbative (linear regime)

- **Ψ4 Extraction**
  - ✓ 3D Ψ4 extraction consistent with Perturbative (linear regime)

Comparison with perturbative simulations indicates that both methods must be taken into account to extract accurate waveforms.
Conclusions (cont.)

- **Quadrupole Extraction**
  - ✓ Frequencies are properly captured
  - ✓ Amplitudes are underestimated
  - ✓ BEST: SQF2

- **Non-linear effects**
  - ✓ radial couplings
  - ✓ overtones couplings
  - ✓ self couplings

[Shibata Sekiguchi 2003]

[Passamonti et al. 2006 / Dimmelmeier et al 2007]
Thank you very much!

REFERENCES:
1. Nagar 2004
2. gr-.qc/0408041 2004
3. Nagar et al. 2004
4. Bernuzzi, Nagar & De Pietri 2008
5. Bernuzzi & Nagar 2008